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LETTERS TO THE EDITOR

A NOTE ON THE PROPAGATION OF SOUND THROUGH CAPILLARY TUBES WITH MEAN FLOW

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In their paper Jeong and Ih [1] have investigated the sound propagation through a capillary duct with mean flow. Using the general formulations of conservation equations with non-isentropic conditions based on the low reduced frequency solution they have solved the linearized governing equations with the recursive use of numerical methods and numerical solutions were obtained up to high shear wave numbers.

In this note, the following first order approximate linearized governing equations which have been obtained by Jeong and Ih [1] are used:

$$k(\mathrm{i}\rho + \Gamma u + M_0\Gamma\rho) + \mathrm{d}v/\mathrm{d}\eta + v/\eta = 0, \tag{1}$$

$$\mathrm{i}u + M_0\Gamma u - (4\bar{M}/k)\eta v = -(\Gamma/\gamma)p + (1/s^2)(\mathrm{d}^2u/\mathrm{d}\eta^2 + (1/\eta)\,\mathrm{d}u/\mathrm{d}\eta), \tag{2}$$

$$\mathrm{i}T + M_0\Gamma T = (1/\sigma^2 s^2)(\mathrm{d}^2T/\mathrm{d}\eta^2 + (1/\eta)\,\mathrm{d}T/\mathrm{d}\eta) + ((\gamma-1)/\gamma)(\mathrm{i} + M_0\Gamma)p$$
$$- (8\bar{M}(\gamma-1)/s^2)(\mathrm{d}/\mathrm{d}\eta)(u\eta), \tag{3}$$

$$p = \rho + T, \tag{4}$$

$$\frac{\mathrm{d}p}{\mathrm{d}\eta} = 0, \tag{5}$$

where

$$\bar{M} = \int_0^1 M(\eta)2\eta\,\mathrm{d}\eta, \qquad M_0 = \left(\frac{s^2}{\gamma}\frac{\mathrm{d}p_0}{\mathrm{d}\xi}\right)\left(\frac{1-\eta^2}{4}\right) \equiv 2\bar{M}(1-\eta^2),$$

$$\Gamma = \Gamma_1 + i\Gamma_2, \quad s = R\sqrt{\frac{\bar{\rho}\omega}{\mu}} \quad \eta = r/R, \quad \xi = \omega x/\bar{a}. \tag{6}$$

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Here, u, v represent the perturbed axial and radial velocities, respectively, ρ the perturbed density, Γ means the normalized complex propagation constant [2], s the shear wave number, $\sigma = \sqrt{\mu c_p/\kappa}$ signifies the square root of the Prandtl number, $k = \omega R/\bar{a}$ the reduced frequency, μ the viscosity, κ the thermal conductivity, c_p denotes the specific heat at constant pressure, x, r denote axial and radial co-ordinates, respectively, $\bar{\rho}$, $\bar{a}$ are the average density and speed of sound in the isentropic condition without fluid flow, γ the ratio of specific heats, R the tube radius, p, T are the perturbed pressure and temperature, respectively, and ω the frequency of harmonic acoustic disturbances.

The expression for M_0 in equations (6) means that Poiseuille-like consideration of the motion is given for the steady flow in the axial direction. Since p is assumed to be a radially independent parameter [1] then from equation (2) one easily derives through differentiation with respect to radial co-ordinate η the following equation:

$$\mathrm{i}\frac{\mathrm{d}u}{\mathrm{d}\eta} + M_0\Gamma\frac{\mathrm{d}u}{\mathrm{d}\eta} - 4\bar{M}\Gamma u\eta - (4\bar{M}/k)\frac{\mathrm{d}\eta v}{\mathrm{d}\eta} = \frac{1}{s^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right). \tag{7}$$

By using equation (1), equation (7) can be written as

$$\mathrm{i}\frac{\mathrm{d}u}{\mathrm{d}\eta} + M_0\Gamma\frac{\mathrm{d}u}{\mathrm{d}\eta} - 4\bar{M}\Gamma u\eta + (4\bar{M}/k)\ k\eta(\mathrm{i}\rho + \Gamma u + M_0\Gamma\rho) = (\mathrm{i} + M_0\Gamma)\frac{\mathrm{d}u}{\mathrm{d}\eta}$$
$$+ 4\bar{M}(\mathrm{i} + M_0\Gamma)\eta\rho = \frac{1}{s^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right). \tag{8}$$

Then, from this equation one easily obtains the following expression for the function $\rho(\eta)$:

$$\rho = -\frac{1}{4\bar{M}\eta}\frac{\mathrm{d}u}{\mathrm{d}\eta} + \frac{1}{s^3 4\bar{M}\eta(\mathrm{i} + M_0\Gamma)}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right). \tag{9}$$

Next, from equation (3) one easily obtains the following expression for the function $T(\eta)$:

$$T = -\frac{8\bar{M}(\gamma-1)}{s^2(\mathrm{i} + M_0\Gamma)}\frac{\mathrm{d}}{\mathrm{d}\eta}(u\eta) + \frac{(\gamma-1)}{\gamma}p + \frac{1}{\sigma^2 s^2(\mathrm{i} + M_0\Gamma)\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\right). \tag{10}$$

From equations (9) and (10) and equation (4) a new expression follows:

$$\rho + T = p = -\frac{1}{4\bar{M}\eta}\frac{\mathrm{d}u}{\mathrm{d}\eta} + \frac{1}{s^2 4\bar{M}\eta(\mathrm{i} + M_0\Gamma)}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right) - \frac{8\bar{M}(\gamma-1)}{s^2(\mathrm{i} + M_0\Gamma)}\frac{\mathrm{d}}{\mathrm{d}\eta}(u\eta)$$
$$+ \frac{1}{\sigma^2 s^2(\mathrm{i} + M_0\Gamma)\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\right) + \frac{(\gamma-1)}{\gamma}p. \tag{11}$$

Equation (11) can be written in the form

$$\eta(\mathrm{i} + M_0\Gamma)\frac{p}{\gamma} = -\frac{(\mathrm{i} + M_0\Gamma)}{4\bar{M}}\frac{\mathrm{d}u}{\mathrm{d}\eta} - \frac{8\bar{M}(\gamma-1)\eta}{s^2}\frac{\mathrm{d}}{\mathrm{d}\eta}(u\eta) + \frac{1}{s^2 4\bar{M}}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right)$$
$$+ \frac{1}{\sigma^2 s^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\right). \tag{12}$$

Here it should be remarked that according to the Jeong and Ih analytical model the radial gradient of the pressure fluctuations can be neglected as can be seen in equation (5). Therefore, in equations (2)–(5) the function $p = p(\eta)$ can be regarded as a radially independent parameter ($p = p(\eta) = const.$ at any fixed ξ). This fact can also be found in Jeong and Ih work [1], p. 68, equation (3); p. 70, equations (14a, b). Consequently, the derivation of equation (7) is correct and then derivations of equations (8), (9), (11) and (12) are also correct. This process is not strange because equations (1)–(5) are ordinary differential equations with η as the single independent variable. Now, instead of considering equations (1)–(5), the set of equations (1), (5), (7), (10) and (12) will be considered herein.

Equation (12), when integrated with respect to η from 0 to 1, can be written as

$$\frac{p}{\gamma}\int_0^1 \eta(\mathrm{i} + M_0\Gamma)\,\mathrm{d}\eta = -\frac{1}{4\bar{M}}\int_0^1 (\mathrm{i} + M_0\Gamma)\frac{\mathrm{d}u}{\mathrm{d}\eta}\mathrm{d}\eta - \frac{8\bar{M}(\gamma-1)}{s^2}\int_0^1 \eta\frac{\mathrm{d}(u\eta)}{\mathrm{d}\eta}\mathrm{d}\eta$$
$$+\frac{1}{4s^2\bar{M}}\int_0^1 \frac{\mathrm{d}}{\mathrm{d}\eta}\left[\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right]\mathrm{d}\eta + \frac{1}{\sigma^2 s^2}\int_0^1 \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\right)\mathrm{d}\eta. \qquad (13)$$

Now, by introducing the expression for M_0 (6) in term on the left-hand side of equation (13), one obtains

$$\frac{p}{\gamma}\int_0^1 \eta(\mathrm{i} + M_0\Gamma)\,\mathrm{d}\eta = \frac{p}{\gamma}\int_0^1 \eta[\mathrm{i} + 2\bar{M}(1-\eta^2)\Gamma]\,\mathrm{d}\eta = \frac{p}{\gamma}\left(\frac{\mathrm{i}}{2} + \bar{M}\Gamma - \frac{2}{4}\bar{M}\Gamma\right) = \frac{p(\mathrm{i} + \bar{M}\Gamma)}{2\gamma}. \qquad (14)$$

Similarly, using the boundary condition given in reference [1], $u = 0$ at $\eta = 1$, taking the expression for M_0 (6) into account and integrating by parts the first term on the right-hand side of equation (13), gives

$$-\frac{1}{4\bar{M}}\int_0^1 [\mathrm{i} + 2\bar{M}\Gamma(1-\eta^2)]\frac{\mathrm{d}u}{\mathrm{d}\eta}\mathrm{d}\eta = \left(\frac{\mathrm{i}}{4\bar{M}} + \frac{\Gamma}{2}\right)u_{|\eta=0} + \frac{\Gamma}{2}\int_0^1 \eta^2\frac{\mathrm{d}u}{\mathrm{d}\eta}\mathrm{d}\eta = \frac{\mathrm{i} + 2\bar{M}\Gamma}{4\bar{M}}u_{|\eta=0}$$
$$+\frac{\Gamma}{2}(1^2 u_{|\eta=1} - 0^2 u_{|\eta=0}) - \frac{\Gamma}{2}\int_0^1 2\eta u\,\mathrm{d}\eta = \frac{\mathrm{i} + 2\bar{M}\Gamma}{4\bar{M}}u_{|\eta=0} - \frac{\Gamma}{2}\langle u\rangle, \qquad (15)$$

where $\langle u\rangle$ means the sectional average in radial co-ordinate represented as $\int_0^1 2\eta u\,\mathrm{d}\eta$.

Obviously, $\langle u\rangle$ is the volume–velocity fluctuation. In the same way, by integrating the second term on the right-hand side of equation (13) by parts and using boundary condition given in reference [1] , $u = 0$ at $\eta = 1$, one obtains

$$-\frac{8\bar{M}(\gamma-1)}{s^2}\int_0^1 \eta\frac{\mathrm{d}(u\eta)}{\mathrm{d}\eta}\mathrm{d}\eta = -\frac{8\bar{M}(\gamma-1)}{s^2}(1^2 u_{|\eta=1} - 0^2 u_{|\eta=0}) + \frac{4\bar{M}(\gamma-1)}{s^2}\int_0^1 2\eta u\,\mathrm{d}\eta$$
$$= \frac{4\bar{M}(\gamma-1)}{s^2}\langle u\rangle. \qquad (16)$$

Next, integrating the third term on the right-hand side of equation (13) and using the boundary condition given in reference [1], $\mathrm{d}u/\mathrm{d}\eta = 0$ at $\eta = 0$, gives

$$\frac{1}{4\bar{M}s^2}\int_0^1 \frac{\mathrm{d}}{\mathrm{d}\eta}\left[\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right]\mathrm{d}\eta$$

$$= \frac{1}{4\bar{M}s^2}\left[\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right]_{|\eta=1} - \frac{1}{4\bar{M}s^2}\left[\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right]_{|\eta=0}$$

$$= \operatorname*{Lim}_{\eta\to 1}\left\{\frac{1}{4\bar{M}s^2}\frac{1}{\eta}\left[\frac{\mathrm{d}u}{\mathrm{d}\eta} + \eta\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}\right]\right\} - \operatorname*{Lim}_{\eta\to 0}\left\{\frac{1}{4\bar{M}s^2}\frac{1}{\eta}\left[\frac{\mathrm{d}u}{\mathrm{d}\eta} + \eta\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}\right]\right\}$$

$$= \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1} + \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} - \operatorname*{Lim}_{\eta\to 0}\left\{\frac{1}{4\bar{M}s^2}\left[\frac{\mathrm{d}u(\eta)/\mathrm{d}\eta - (\mathrm{d}u/\mathrm{d}\eta_{|\eta=0} = 0)}{\eta - 0}\right]\right\}$$

$$- \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0} = \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1} + \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} - \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0} - \frac{1}{4\bar{M}s^2}\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0}$$

$$= \frac{1}{4\bar{M}s^2}\left(\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} + \frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1} - 2\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0}\right). \tag{17}$$

Finally, by integrating with respect to η from 0 to 1 and using the boundary condition given in reference [1], $\mathrm{d}T/\mathrm{d}\eta = 0$ at $\eta = 0$, the fourth term on the right-hand side of equation (13) becomes

$$\frac{1}{\sigma^2s^2}\int_0^1 \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\right)\mathrm{d}\eta = \frac{1}{\sigma^2s^2}1\frac{\mathrm{d}T}{\mathrm{d}\eta}_{|\eta=1} - \frac{1}{\sigma^2s^2}0\frac{\mathrm{d}T}{\mathrm{d}\eta}_{|\eta=0} = \frac{1}{\sigma^2s^2}\frac{\mathrm{d}T}{\mathrm{d}\eta}_{|\eta=1}. \tag{18}$$

Thus, from equations (13)–(18), the acoustic pressure p can be expressed as

$$p = \frac{\gamma}{2\bar{M}s^2(\mathrm{i} + \bar{M}\Gamma)}\left(\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} + \frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1} - 2\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0}\right) + \frac{\gamma(\mathrm{i} + 2\bar{M}\Gamma)}{2\bar{M}(\mathrm{i} + \bar{M}\Gamma)}u_{|\eta=0} - \frac{\gamma\Gamma}{\mathrm{i} + \bar{M}\Gamma}\langle u\rangle$$

$$+ \frac{2\gamma}{\sigma^2s^2(\mathrm{i} + \bar{M}\Gamma)}\frac{\mathrm{d}T}{\mathrm{d}\eta}_{|\eta=1} + \frac{8\bar{M}\gamma(\gamma - 1)}{s^2(\mathrm{i} + \bar{M}\Gamma)}\langle u\rangle. \tag{19}$$

Similarly, averaging equation (7) yields

$$\int_0^1\left[\mathrm{i}\frac{\mathrm{d}u}{\mathrm{d}\eta} + M_0\Gamma\frac{\mathrm{d}u}{\mathrm{d}\eta} - 4\bar{M}\Gamma u\eta - (4\bar{M}/k)\frac{\mathrm{d}(\eta v)}{\mathrm{d}\eta}\right]\mathrm{d}\eta$$

$$= \int_0^1\left[\mathrm{i}\frac{\mathrm{d}u}{\mathrm{d}\eta} + 2\bar{M}\Gamma(1 - \eta^2)\frac{\mathrm{d}u}{\mathrm{d}\eta} - 4\bar{M}\Gamma u\eta\right]\mathrm{d}\eta$$

$$- \frac{4\bar{M}}{k}\int_0^1 \frac{\mathrm{d}(\eta v)}{\mathrm{d}\eta}\mathrm{d}\eta = (\mathrm{i} + 2\bar{M}\Gamma)\int_0^1 \frac{\mathrm{d}u}{\mathrm{d}\eta}\mathrm{d}\eta - 2\bar{M}\Gamma\int_0^1\left(\eta^2\frac{\mathrm{d}u}{\mathrm{d}\eta} + 2\eta u\right)\mathrm{d}\eta$$

$$= -(\mathrm{i} + 2\bar{M}\Gamma)u_{|\eta=0} - 2\bar{M}\Gamma\int_0^1 \frac{\mathrm{d}(\eta^2u)}{\mathrm{d}\eta}\mathrm{d}\eta = -(\mathrm{i} + 2\bar{M}\Gamma)u_{|\eta=0}$$

$$= \frac{1}{s^2}\int_0^1 \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right)\mathrm{d}\eta = \frac{1}{s^2}\left(\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} + \frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1} - 2\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0}\right). \tag{20}$$

In order to demonstrate that differentiating equation (2) and then integrating equation (7) results in a quite coincidence of original (2) and final (7) equations, rewrite equation (2) as follows:

$$-(\Gamma/\gamma)p = \mathrm{i}u + M_0\Gamma u - (4\bar{M}/k)\eta v - (1/s^2)(\mathrm{d}^2u/\mathrm{d}\eta^2 + (1/\eta)\,\mathrm{d}u/\mathrm{d}\eta)$$

$$= \mathrm{i}u + 2\bar{M}\Gamma u - 2\bar{M}\Gamma\eta^2 u - (4\bar{M}/k)\,\eta v - \frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right). \quad (21)$$

Since $-(\Gamma/\gamma)p$ is the radially independent parameter, it can be determined by the value of the expression on the right-hand side of equation (21) at a arbitrary value of η, for example, at $\eta = 0$.

Then,

$$-(\Gamma/\gamma)p = \left[\mathrm{i}u + 2\bar{M}\Gamma u - 2\bar{M}\Gamma\eta^2 u - (4\bar{M}/k)\,\eta v - \frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u}{\mathrm{d}\eta}\right)\right]_{\eta=0}. \quad (22)$$

If both sides of equation (7) are integrated over the interval 0 to η, it becomes for any η

$$\mathrm{i}u(\eta) + M_0\Gamma u(\eta) - (4\bar{M}/k)\eta v(\eta) - [\mathrm{i}u(\eta) + M_0\Gamma u(\eta) - (4\bar{M}/k)\eta v(\eta)]_{\eta=0}$$

$$= \frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u(\eta)}{\mathrm{d}\eta}\right) - \left[\frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u(\eta)}{\mathrm{d}\eta}\right)\right]_{\eta=0}. \quad (23)$$

By using equation (22), equation (23) can be written as

$$\mathrm{i}u(\eta) + M_0\Gamma u(\eta) - (4\bar{M}/k)\eta v(\eta) = [\mathrm{i}u(\eta) + M_0\Gamma u(\eta) - (4\bar{M}/k)\eta v(\eta)]_{\eta=0}$$

$$-\left[\frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u(\eta)}{\mathrm{d}\eta}\right)\right]_{\eta=0} + \frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u(\eta)}{\mathrm{d}\eta}\right) = -(\Gamma/\gamma)p + \frac{1}{s^2}\frac{1}{\eta}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}u(\eta)}{\mathrm{d}\eta}\right). \quad (24)$$

It can readily be seen that equation (24) coincides with equation (2).

Equation (20) can be rewritten in the form

$$(\mathrm{i} + 2\bar{M}\Gamma)u_{|\eta=0} = -\frac{1}{s^2}\left(\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=1} - 2\frac{\mathrm{d}^2u}{\mathrm{d}\eta^2}_{|\eta=0} + \frac{\mathrm{d}u}{\mathrm{d}\eta}_{|\eta=1}\right). \quad (25)$$

In deriving equation (25), the boundary conditions given in reference [1], $u = 0$, $v = 0$ at $\eta = 1$ and $\mathrm{d}u/\mathrm{d}\eta = 0$ at $\eta = 0$, are used.

The resulting form of equation (13) may be simplified by substituting equation (25) into equation (19). Equation (19) is then written as

$$p = -\frac{\gamma\Gamma}{\mathrm{i} + \bar{M}\Gamma}\langle u\rangle + \frac{2\gamma}{\sigma^2 s^2(\mathrm{i} + \bar{M}\Gamma)}\frac{\mathrm{d}T}{\mathrm{d}\eta}_{|\eta=1} + \frac{8\bar{M}\gamma(\gamma-1)}{s^2(\mathrm{i} + \bar{M}\Gamma)}\langle u\rangle. \quad (26)$$

In order to investigate the validity of the obtained formula (26) the acoustic impedance $Z = (1/\gamma)\,p/\langle u\rangle$ was found and compared with the result of Jeong and Ih [1] for the case of high shear wave numbers. In this case, formula (26) takes the form

$$p = -\frac{\gamma\Gamma}{(\mathrm{i} + \bar{M}\Gamma)}\langle u\rangle. \quad (27)$$

According to Jeong and Ih [1, p. 75], the real part of the propagation constant becomes zero, the imaginary part of the propagation constant of forward and backward waves tends to approach the value of $1/(1 \pm \bar{M})$, respectively, and the acoustic impedance approaches 1, as the shear wave number becomes high.

Then, by substituting $\Gamma = -\,\mathrm{i}/(1 + \bar{M})$ into equation (27), one can obtain for the forward propagating wave,

$$Z = \frac{p}{\gamma\langle u\rangle} = -\frac{\mathrm{i}(1+\bar{M})}{(1+\bar{M})(-\,\mathrm{i} - \mathrm{i}\bar{M} + \mathrm{i}\bar{M})} = 1.$$

Similarly, substituting $\Gamma = \mathrm{i}/(1 - \bar{M})$ into equation (27), gives for the backward propagating wave

$$Z = \frac{p}{\gamma\langle u\rangle} = \frac{\mathrm{i}(1-\bar{M})}{(1-\bar{M})(\mathrm{i} - \mathrm{i}\bar{M} + \mathrm{i}\bar{M})} = 1.$$

These results coincide with those of Jeong and Ih [1], thus validating the present mathematical solution.

It should be noted that if $\Gamma = 1/\mathrm{i}\bar{M}$ then as can be seen in equation (27) $Z = \infty$. This means that the physically realizable axial distribution of the volume–velocity fluctuations with $\Gamma = 1/\mathrm{i}\bar{M}$ does not exist because in the case the source of sound is desired to produce the pressure fluctuations with the unbounded amplitudes. Consequently, for the case of high shear wave numbers, the acoustic pressure p remains stable.

Formula (26) would be useful in the testing of the solutions obtained by various numerical methods.

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